MaPS 2024 Senior Syllabus and Schedule

Most topics are **problem-solving topics** which will help develop your problem solving skills and will be applicable in mathematical competitions. However, seniors will cover some **extension topics** which are designed to give you a taste of further research-level mathematics. These will be clearly stated in the syllabus.

Term 1

Bounding Arguments

Understanding the long term trends for how mathematical expressions behave is a key step in solving many problems in number theory and algebra. **Bounding Arguments** will introduce some key concepts such as **dominating terms** and **consecutive squares** and their applications to problem solving questions.

Geometric Transformations

At first glance, geometry and geometric problems can seem like a collection of results about a fixed collection of shapes and lines on a plane. However, they can often be interpreted as objects that are related to each other through **Geometric Transformations**. This innovation can bring much deeper insight and understanding to geometry. This topic will introduce and explore the ideas of **dilation**, **rotation**, **translation** and **reflection**.

Harder Modular Arithmetic

As an extension to the **Modular Arithmetic** topic covered in the junior stream, **Harder Modular Arithmetic** will extend the idea of looking at remainders of numbers after division by a certain modulus. This topic will cover **Euler's Phi Function**, **Euler's Theorem** and **Primitive Roots** and apply these to more advanced problems.

Ramsey Theory

Ramsey Theory is a branch of mathematics that focuses on the appearance of order in a substructure given a structure of a known size. Perhaps you heard about the following fact which is one of the most basic results in Ramsey Theory: among any group of 6 people there must be either three people who are friends to each other, or three people who are not friends to each other. You will see how this and harder similar results can be formulated and verified in terms of **coloured graphs**. Problems there become very difficult very quickly with many of them still being open. This is an **extension topic**.

Term 2

Harder Inequalities

As an extension to **Inequalities** in the junior stream, **Harder Inequalities** will build on the fundamental ideas about inequalities to introduce more advanced inequalities like the **Cauchy-Schwarz Inequality**, **Weighted AM-GM** and **Jensen's Inequality**.

Collinearity and Concurrency

An important concept in geometry is the idea that multiple lines can pass through the same point, or multiple points can lie on the same line. Although this can seem obvious, they can be well hidden in geometry diagrams! This topic will explore techniques to prove **Collinearity and Concurrency** including **Ceva's Theorem** and **Menelaus' Theorem**.

Game Theory

Game Theory is a mathematical framework to study the strategy of rational decision makers in well defined games. This can be used to prove that no winning strategy exists in games like Naughts and Crosses or Checkers, but can also be used to prove that a winning strategy does exist in games like Connect Four. This topic will explore the key concepts in game theory and apply them to basic mathematical problems.

Pell's Equation

Pell's Equations are Diophantine Equations of the form $x^2 - ny^2 = 1$. Although they appear simple, their solutions may be very tricky to find! In some cases, the first non-trivial solution to such an equation can be really huge. For example, try to find at least one solution in positive integers of $x^2-13y^2 = 1$ without computer assistance. In this unit you will get an insight on how to solve Pell's equations and explore many fascinating properties of their solutions. This is an **extension topic**.

Term 3

Functions and Functional Equations

Functions are an important and simple concept in mathematics which links the elements of two sets together. These functions can have a large variety of properties which will be explored in this topic. This will also introduce the idea of a **functional equation**, which as it sounds, is an equation that links functions together, and we are interested in finding all functions that satisfy such equations.

Harder Graph Theory

This unit extends the fundamental ideas introduced in **Graph Theory** from the Junior Stream. **Harder Graph Theory** will cover topics such as **cliques** and **colourings** which in fact leads to many open areas of research!

Recurrence Relations

Recurrence Relations can be thought of as an equation that recursively generates a sequence of numbers. Perhaps the most canonical example are the famous Fibonacci numbers which are defined by $f_1 = f_2 = 1$ and $f_{n+2} = f_{n+1} + f_n$ for $n \geq 1$. This is often how Fibonacci numbers are presented, but this topic will cover the theory of recurrence relations to find a closed formula for each Fibonacci number and generalise this to other re-

currence relations! This is an **extension topic**.

Group Theory

When we are first introduced to sets of numbers like the Naturals, Integers, Rationals and Reals, we often take their properties for granted, such as **commutativity**, **associativity** and **distributivity**. **Group Theory** is a field of study that generalises these properties to abstract objects which reveals insight to the structure of these sets that allows arithmetic to hold. Indeed, such generalisations have found applications in fields as diverse as music, chemistry and physics! This is an **extension topic**.

Term 4

Mathematical Talk Competition

This is an **optional** competition that takes place in late November that will mark the end of the Correspondence Program for the year. You will have the opportunity to work in a group to research a **mathematical topic** and present a 5-10 minute mathematical talk. Further instructions and a list of available topics will be provided at the end of term 3.

You will be given 8 weeks to do the required research, prepare and practice their talk with the guidance of a mentor. **Prizes** may be awarded to the best talk.

Draft Schedule

Term 1

Date	Event
30 January	Term 1 Begins
12 February	Bounding Arguments: notes and problems posted
25 February	Bounding Arguments: problems due
26 February	Geometric Transformations: notes and problems posted
10 March	Geometric Transformations: problems due
11 March	Harder Modular Arithmetic: notes and problems posted
24 March	Harder Modular Arithmetic: problems due
25 March	Ramsey Theory: notes and problems posted
8 April	Ramsey Theory: problems due
12 April	Term 1 Ends

Term 2

Date	Event
29 April	Term 2 Begins
29 April	Harder Inequalities: notes and problems posted
May	Australian Training Tournament
12 May	Harder Inequalities: problems due
13 May	Collinearity and Concurrency: notes and problems posted
26 May	Collinearity and Concurrency: problems due
27 May	Game Theory: notes and problems posted
9 June	Game Theory: problems due
10 June	Pell's Euqation: notes and problems posted
23 June	Pell's Euqation: problems due
5 July	Term 2 Ends

Term 3

Date	Event
22 July	Term 3 Begins
22 July	Functions and Functional Equations: Notes and Problems posted
4 August	Functions and Functional Equations: Problems due
5 August	Harder Graph Theory: notes and problems posted
18 August	Harder Graph Theory: problems due
19 August	Recurrence Relations: notes and problems posted
1 September	Recurrence Relations: problems due
2 September	Group Theory: notes and problems posted
12 September	AIMO
15 September	Group Theory: problems due
16 September	Face to face session and Mathematical Talk topics assigned
27 September	Term 3 Ends

Term 4

Date	Event
13 October	Mathematical Talk Outline Due
14 October	Term 4 Begins
29 October	Mathematical Talk First Draft Due
10 November	Mathematical Talk Final Draft Due
November	Mathematical Talk Competition and program close
November/December	Australian Training Tournament
20 December	Term 4 Ends